# MECH 401
# Mechanical Design Applications

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# Design Considerations

- Stress
- Deflection
- Strain
- Stiffness
- Stability

Often the controlling factor

- Stress and strain relationships can be studied with Mohr's circle

# Deflection

- When loads are applied, we have deflection
- Depends on
  - Type of loading
    - Tension
    - Compression
    - Bending
    - Torsion
  - Cross-section of member
  - Comparable to pushing on a spring
- We can calculate the amount of beam deflection by various methods

# Superposition

- Determine effects of individual loads separately and add the results
- Tables are useful – see A-9
- May be applied if
  - Each effect is linearly related to the load that produces it
  - A load does not create a condition that affects the result of another load
  - Deformations resulting from any specific load are not large enough to appreciably alter the geometric relations of the parts of the structural system

# Deflection

- There are situations where the tables are insufficient
- We can use energy-methods in these circumstances
- Define strain energy
  - $$U = \int_0^{x_1} F dx$$
- Define strain energy density**
  - $$\mu = \frac{dU}{dV}$$ V – volume
  - Put in terms of σ, ε
  - $$\mu = \int_0^{\varepsilon_x} \sigma_x d\varepsilon_x$$

$$\sigma_x = E\varepsilon_x$$

$$\sigma_x d\varepsilon_x = E\varepsilon_x d\varepsilon_x$$

$$\mu = \frac{1}{2}\sigma_x \varepsilon_x = \frac{1}{2}\frac{\sigma_x{}^2}{E}$$

$$\mu = \frac{dU}{dV}$$

$$\mu dV = dU$$

$$U = \int \left( \frac{1}{2}\frac{\sigma_x{}^2}{E} \right) dV$$

# Example – beam in bending

$$\sigma = \frac{My}{I}$$

$$U = \int \frac{\sigma_x^{\ 2}}{2E} dV$$

$$I = \int y^2 dA$$

$$U = \int \frac{M^2 y^2}{2EI^2} dV$$

$$U = \int \frac{M^2 y^2}{2EI^2} dV = \int \frac{M^2 y^2}{2EI^2} (dA dx) = \int \frac{M^2 \left( \int y^2 dA \right)}{2EI^2} dx$$

$$dV = dA dx$$

$$U = \int \frac{M^2}{2EI} dx$$

$$\frac{M^2}{2EI^2} = f(x)$$

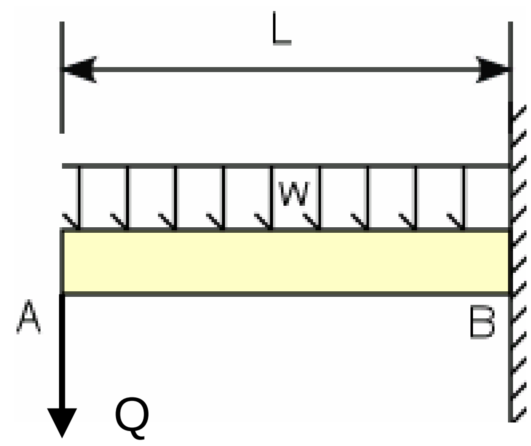
# Castigliano's Theorem

- Deflection at any point along a beam subjected to n loads may be expressed as the partial derivative of the strain energy of the structure WRT the load at that point

$$\delta_i = \frac{\partial U}{\partial F_i}$$

- We can derive the strain energy equations as we did for bending
- Then we take the partial derivative to determine the deflection equation
- Plug in load and solve!
- If there is no load acting at the point of interest, add a dummy load Q, work out equations, then set Q = 0

# Castigliano Example




- Beam AB supports a uniformly distributed load w. Determine the deflection at A.

- No load acting specifically at point A!
  - Apply a dummy load Q

- Substitute expressions for M, ☆M/ ☆ $Q_A$, and $Q_A$ (=0)

- We directed $Q_A$ downward and found ☆ $_A$ to be positive
  - Defection is in same direction as $Q_A$ (downward)

$$\delta_A = \frac{\partial U}{\partial Q_A} = \int_0^L \frac{M}{EI}\left(\frac{\partial M}{\partial Q_A}\right)dx$$

$$M(x) = -Q_A x - \tfrac{1}{2} w x^2$$

$$\frac{\partial M}{\partial Q_A} = -x$$

$$\delta_A = \frac{1}{EI}\int_0^L \left(-\tfrac{1}{2} w x^2\right)\left(-x\right)dx = \frac{wL^4}{8EI}$$

$$\delta_A = \frac{wL^4}{8EI}$$

# Stability

- Up until now, 2 primary concerns
  - Strength of a structure
    - It's ability to support a specified load without experiencing excessive stress
  - Ability of a structure to support a specified load without undergoing unacceptable deformations

Material failure

- Now, look at STABILITY of the structure
  - It's ability to support a load without undergoing a sudden change in configuration

# Buckling

- Buckling is a mode of failure that does not depend on stress or strength, but rather on structural <u>stiffness</u>
- Examples:

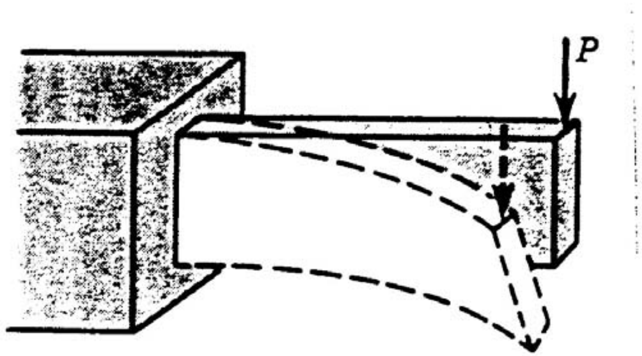



**Fig. 9.6** Twist-bend buckling of a deep, narrow beam.

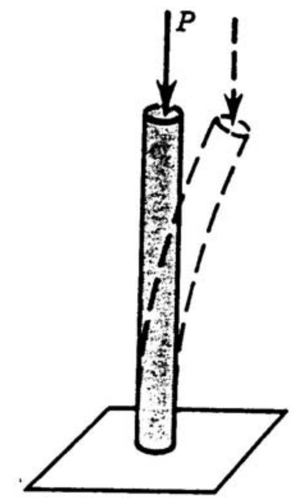



**Fig. 9.7** Buckling of a column under a compressive load.

# More buckling examples…

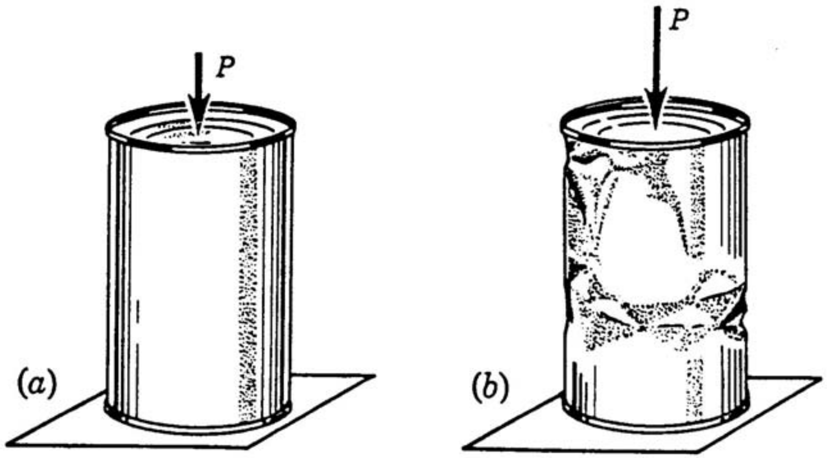



**Fig. 9.8** Buckling and crumpling of the cylindrical walls of a can subjected to compressive force.

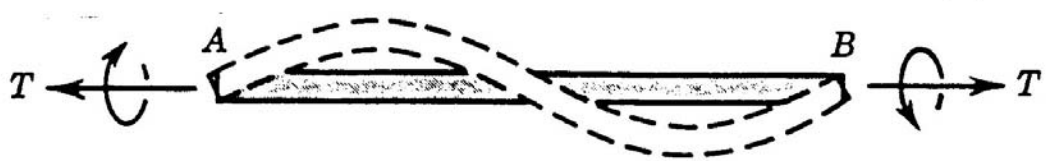


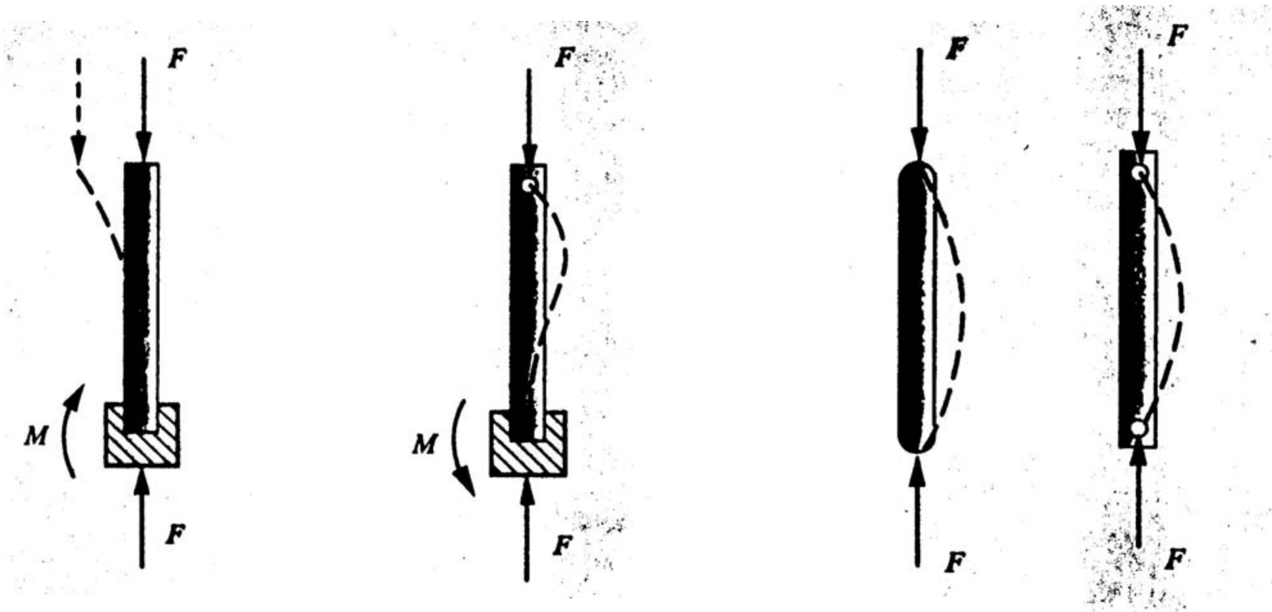

**Fig. 9.9** Twist-bend buckling of a shaft in torsion.

# Buckling

- The most common problem involving buckling is the design of columns
  - Compression members
- The analysis of an element in buckling involves establishing a differential equation(s) for beam deformation and finding the solution to the ODE, then determining which solutions are stable
- Euler solved this problem for columns

# Euler Column Formula

- $$P_{crit} = \frac{c\pi^2 EI}{L^2}$$ $$P_{crit} = \frac{\pi^2 EI}{L_e^2}$$
- Where C is as follows:




C = ¼ ;Le=2L
Fixed-free

C = 2
Fixed-pinned

C = 1
Rounded-rounded
Pinned-pinned

C = 4; Le=L/2
Fixed-fixed

# Buckling

- Geometry is crucial to correct analysis
  - Euler – “long” columns
  - Johnson – “intermediate” length columns
  - Determine difference by slenderness ratio
- The point is that a designer must be alert to the possibility of buckling
- A structure must not only be strong enough, but must also be sufficiently rigid

# Buckling Stress vs. Slenderness Ratio

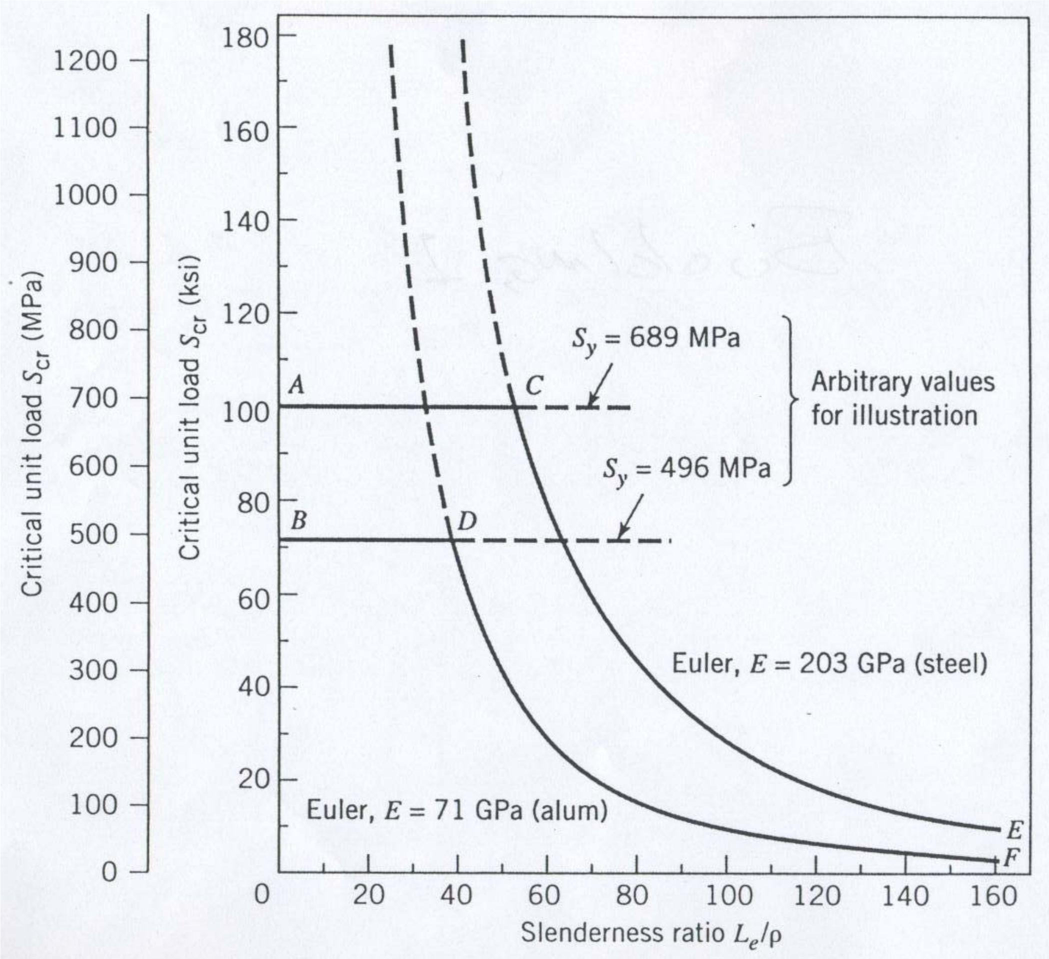



**Figure 5.26**
Euler column buckling curves illustrated for two values of $E$ and $S_y$.

# Johnson Equation for Buckling



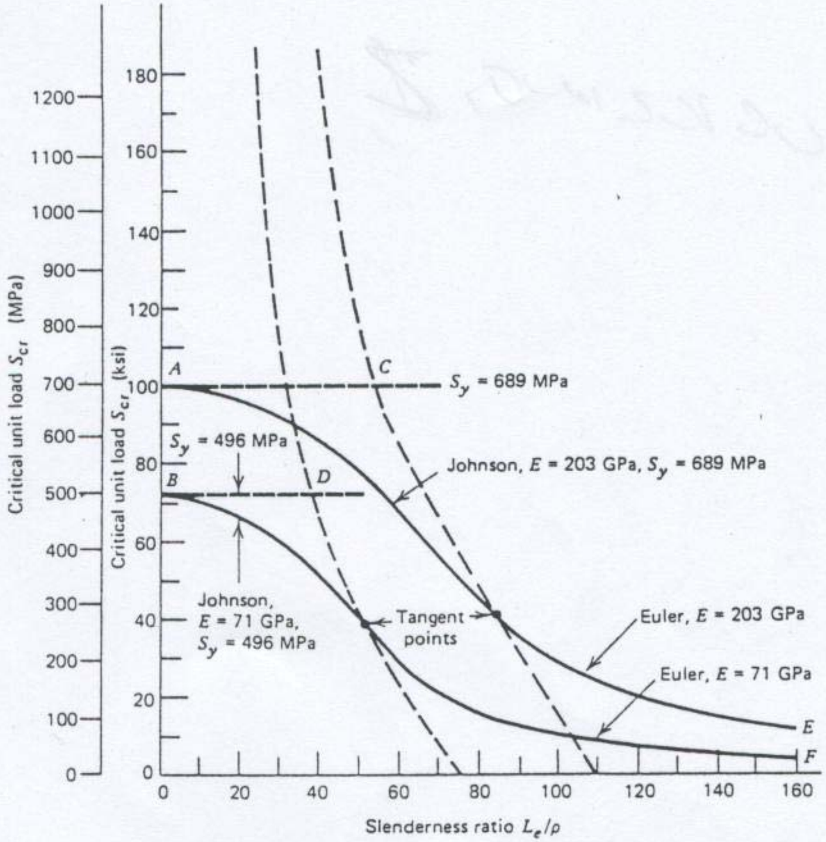



**Figure 5.28** Euler and Johnson column curves illustrated for two values of $E$ and $S_y$ (used in Sample Problems 5.11 and 5.12).

# Solving buckling problems

- Find Euler-Johnson tangent point with $$\frac{L_e}{\rho} = \sqrt{\frac{2\pi^2 E}{S_y}}$$
- For $L_e/\rho$ < tangent point ("intermediate"), use Johnson's Equation: $$S_{cr} = S_y - \frac{S_y{}^2}{4\pi^2 E}\left(\frac{L_e}{\rho}\right)^2$$
- For $L_e/\rho$ > tangent point ("long"), use Euler's equation: $$S_{cr} = \frac{\pi^2 E}{\left(\frac{L_e}{\rho}\right)^2}$$
- For $L_e/\rho < 10$ ("short"), $S_{cr}$ 🕔 $S_y$
- If length is unknown, predict whether it is "long" or "intermediate", use the appropriate equation, then check using the Euler-Johnson tangent point once you have a numerical solution for the critical strength

# Special Buckling Cases

- Buckling in very long Pipe

$$P_{crit} = \frac{c\pi^2 EI}{L^2}$$

Note Pcrit is inversely related to length squared

A tiny load will cause buckling

L = 10 feet vs. L = 1000 feet:

Pcrit1000/Pcrit10 = 0.0001

•Buckling under hydrostatic Pressure

# Pipe in Horizontal Pipe Buckling Diagram

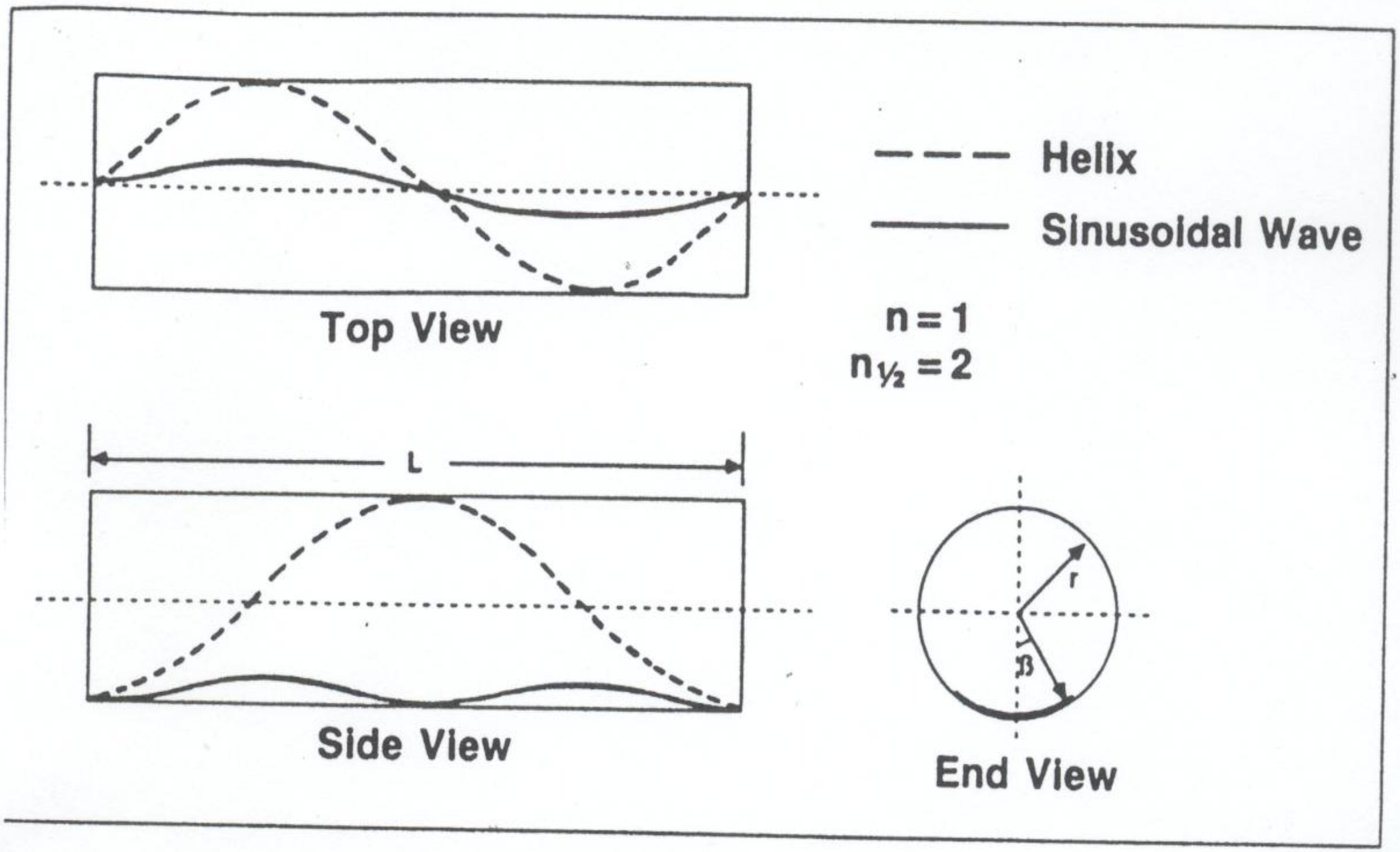
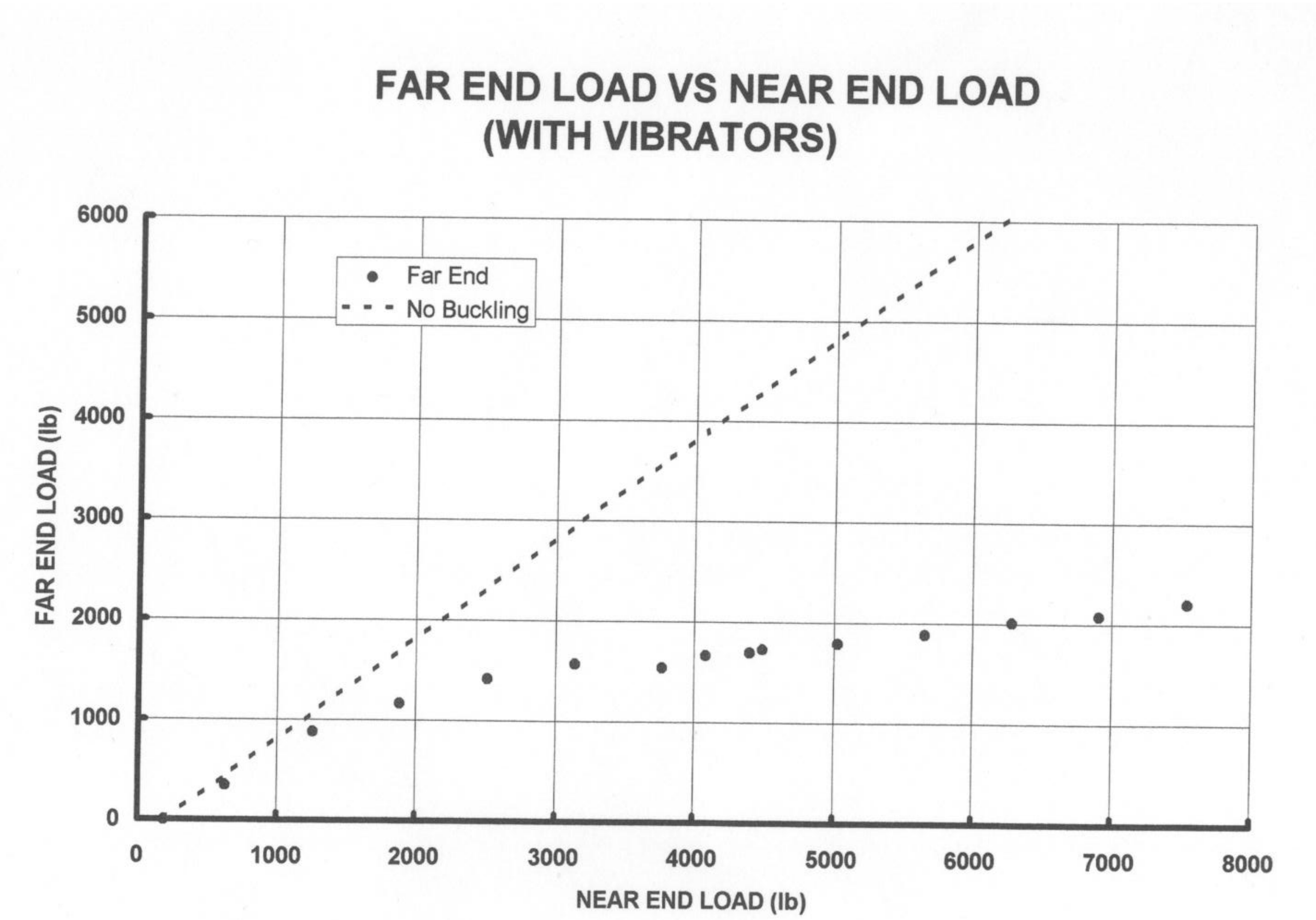
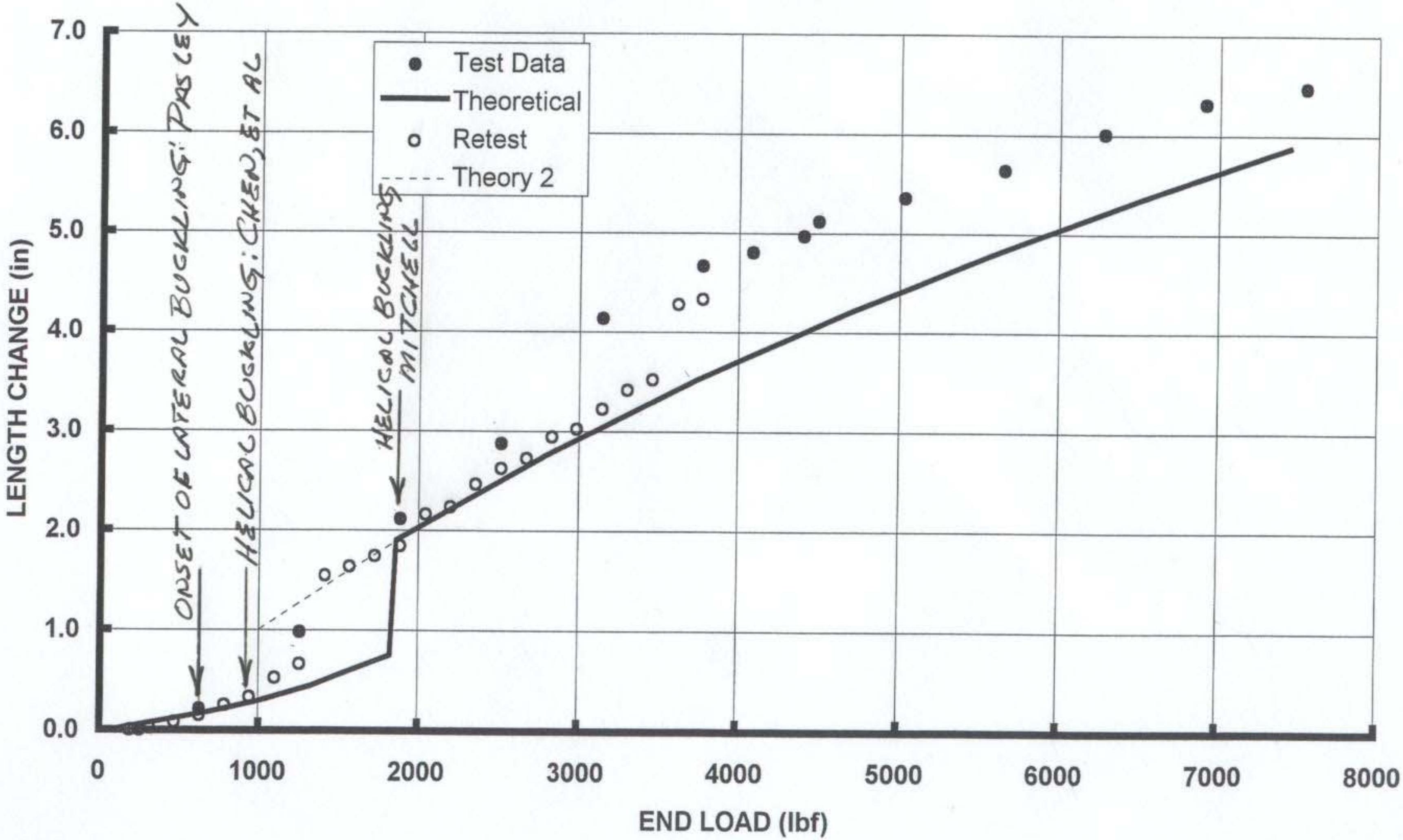
- 


Fig. 1—Postbuckled configuration of pipe in a horizontal hole.

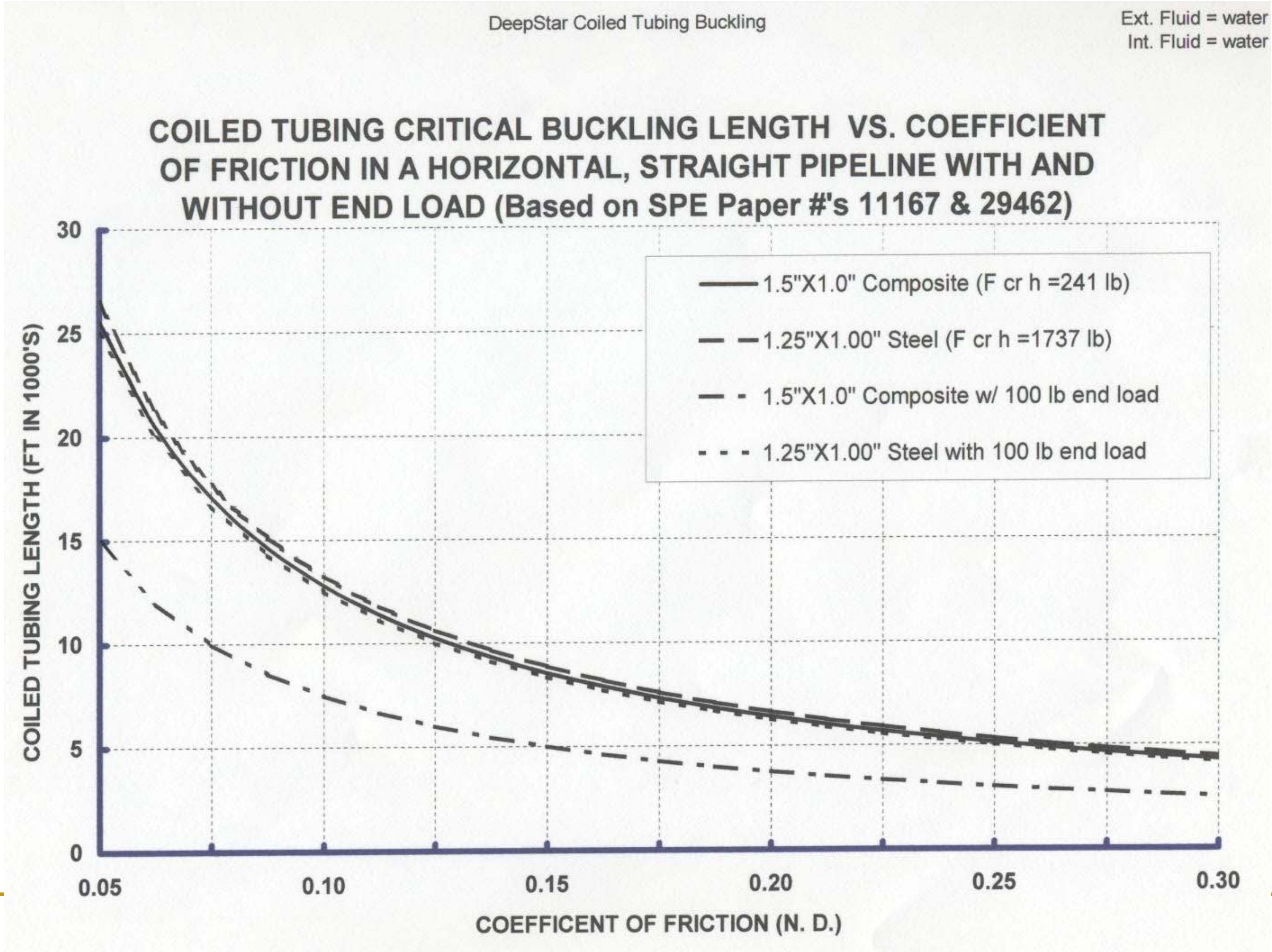
# Far End vs. Input Load with Buckling



# THEORITACAL AND MEASURED LENGTH CHANGE VS. END LOAD

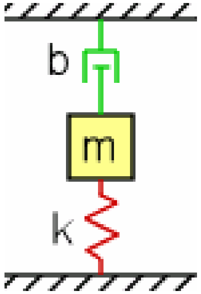


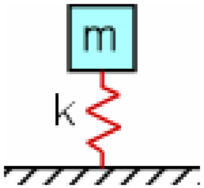
# Buckling Length: Fiberglass vs. Steel

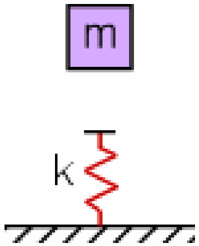


# Impact

- Dynamic loading
  - Impact – Chapter 5
  - Fatigue – Chapter 7
- Shock loading = sudden loading
- Examples?
- 3 categories
  - Rapidly moving loads of constant magnitude
    - Driving over a bridge
  - Suddenly applied loads
    - Explosion, combustion
  - Direct impact
    - Pile driver, jack hammer, auto crash

# Impact, cont.

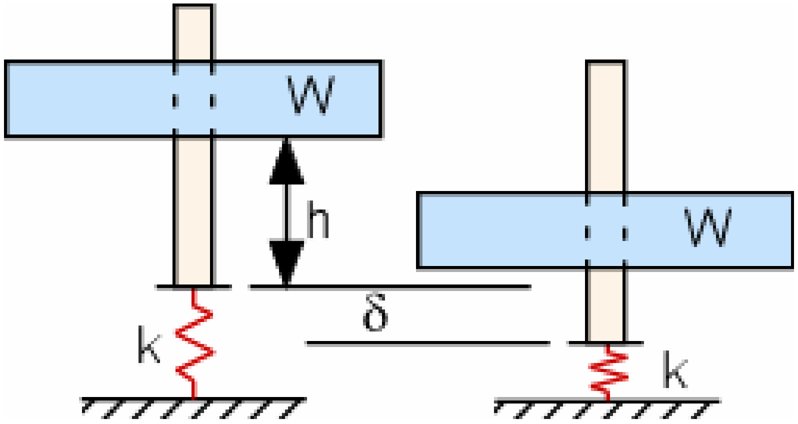
- It is difficult to define the time rates of load application
- Leads to use of empirically determined stress impact factors
- If $\tau$ is time constant of the system, where

$$\tau = 2\pi\sqrt{\frac{m}{k}}$$

  - We can define the load type by the time required to apply the load ($t_{AL}$ = time required to apply the load)
    - Static $t_{AL} > 3\tau$
    - "Gray area" $\frac{1}{2}\tau < t_{AL} < 3\tau$
    - Dynamic $t_{AL} < \frac{1}{2}\tau$

# Stress and deflection due to impact

- W – freely falling mass
- k – structure with stiffness (usually large)
- Assumptions
  - Mass of structure is negligible
  - Deflections within the mass are negligible
  - Damping is negligible
- Equations are only a GUIDE
- h is height of freely falling mass before its release
- $\delta$ is the amount of deflection of the spring/structure



# Impact Assumptions



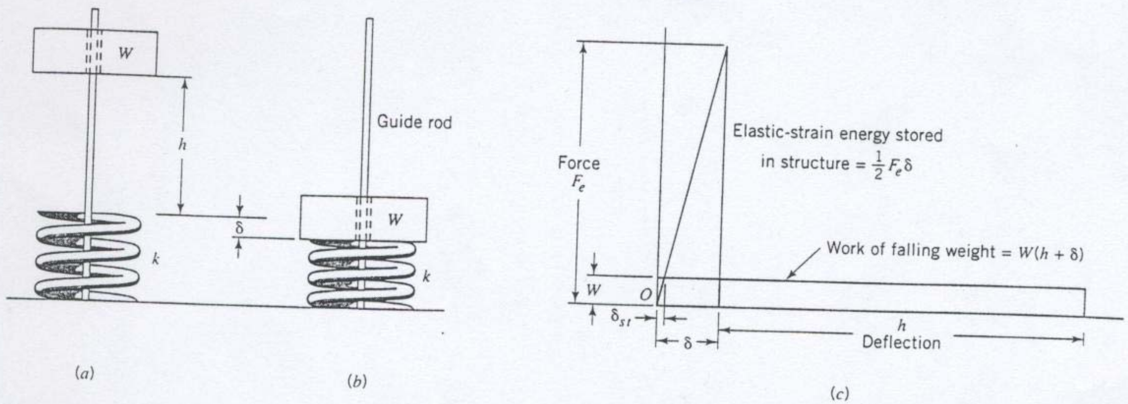



FIGURE 7.3
Impact load applied to elastic structure by falling weight: (*a*) initial position; (*b*) position at instant of maximum deflection; (*c*) force–deflection-energy relationships.

1. The first assumption implies that the dynamic deflection curve (i.e., the instantaneous deflections resulting from impact) is identical to that caused by the static application of the same load, multiplied by an *impact factor*. In reality, the dynamic deflection curve inevitably involves points of higher *local* strain (hence, higher local stress) than does the static curve.
2. *Some* deflection must inevitably occur within the impacting mass itself. To the extent that it does, a portion of the energy is absorbed within the mass, thereby causing the stresses and deflections in the structure to be a little *lower* than the calculated values.
3. Any actual case involves some (though perhaps very little) friction damping in the form of windage, rubbing of the mass on the guide rod and end of the spring (in Figure 7.3), and internal friction within the body of the deflecting structure. This damping can cause the actual stresses and deflections to be significantly less than those calculated from the idealized case.

# Impact Energy Balance

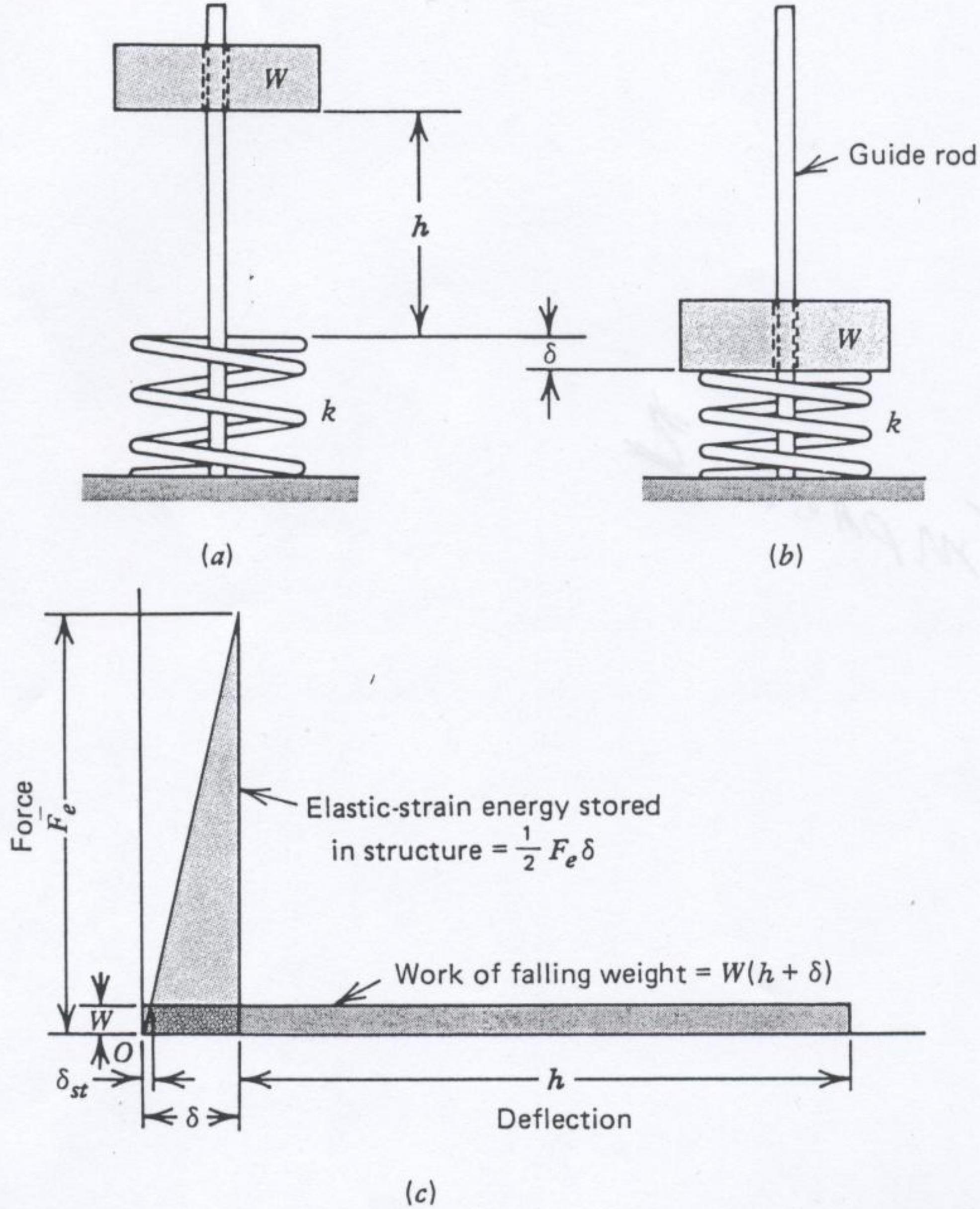
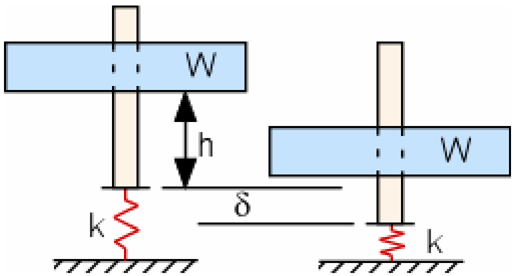



**Figure 7.3** Impact load applied to elastic structure by falling weight: (*a*) initial position; (*b*) position at instant of maximum deflection; (*c*) force–deflection-energy relationships.

# Energy balance




- $F_e$ is the equivalent static force necessary to create an amount of deflection equal to δ
- ½ because spring takes load gradually

$$W(h+\delta)=\frac{1}{2}F_e\delta$$

$$W=k\delta_{static}$$

$$F_e=k\delta$$

$$F_e=\left[\frac{\delta}{\delta_{static}}\right]W$$

$$\frac{F_e}{W}=\frac{\delta}{\delta_s}$$

$$W(h+\delta)=\frac{1}{2}\frac{\delta^2}{\delta_s}W$$

$$h+\delta=\frac{1}{2}\frac{\delta^2}{\delta_s}$$

$$\delta=\delta_s\left(1+\sqrt{1+\frac{2h}{\delta_s}}\right)$$

$$F_e=W\left(1+\sqrt{1+\frac{2h}{\delta_s}}\right)$$

# Impact, cont.

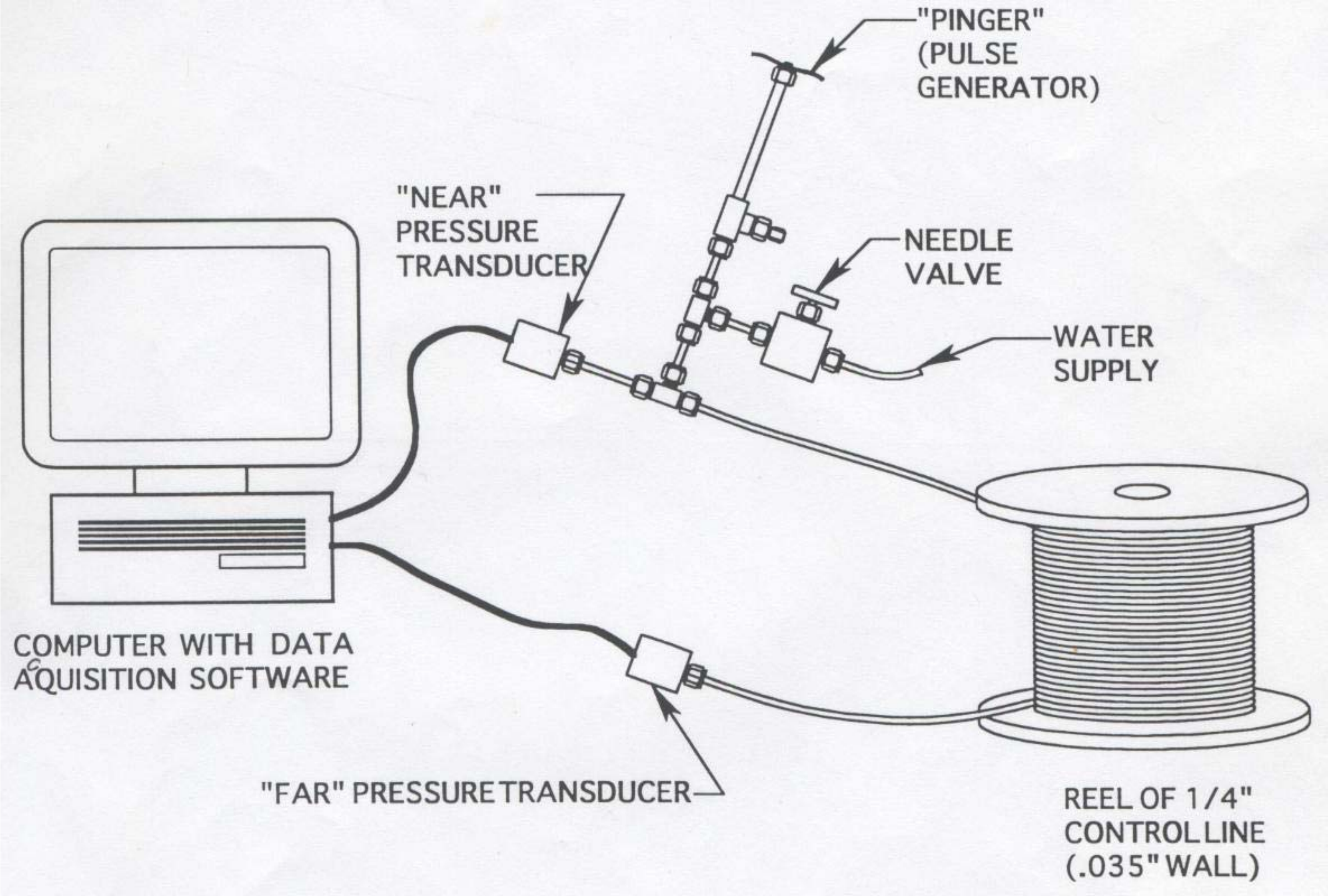
- Sometimes we know velocity at impact rather than the height of the fall
- An energy balance gives:

$$v^2 = 2gh$$

$$\delta = \delta_s \left( 1 + \sqrt{1 + \frac{v^2}{g\delta_s}} \right)$$

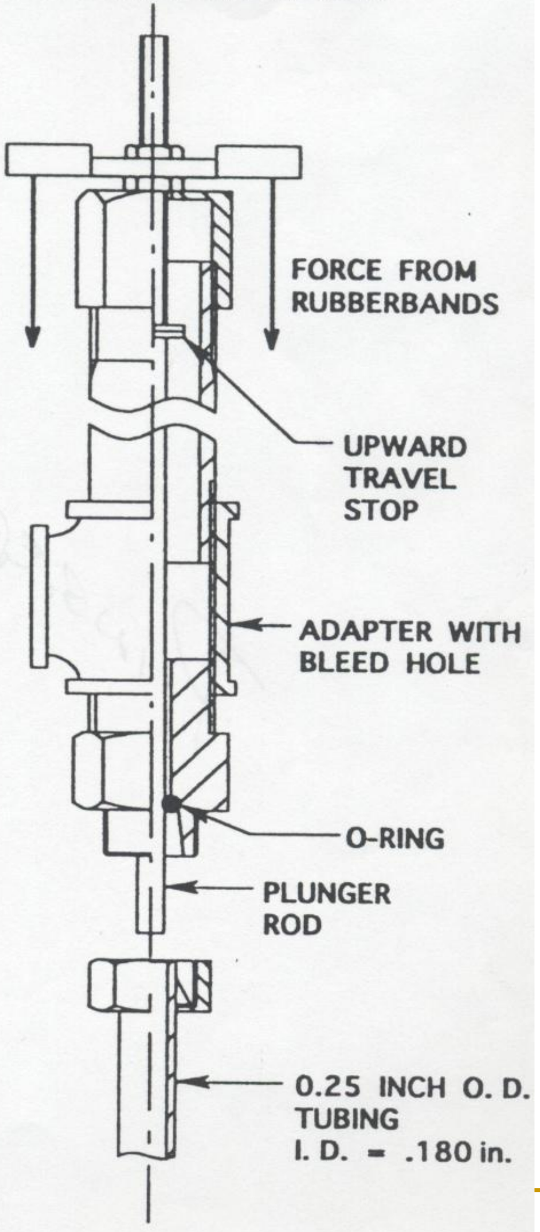
$$F_e = W \left( 1 + \sqrt{1 + \frac{v^2}{g\delta_s}} \right)$$

# Pinger  Pulse Setup

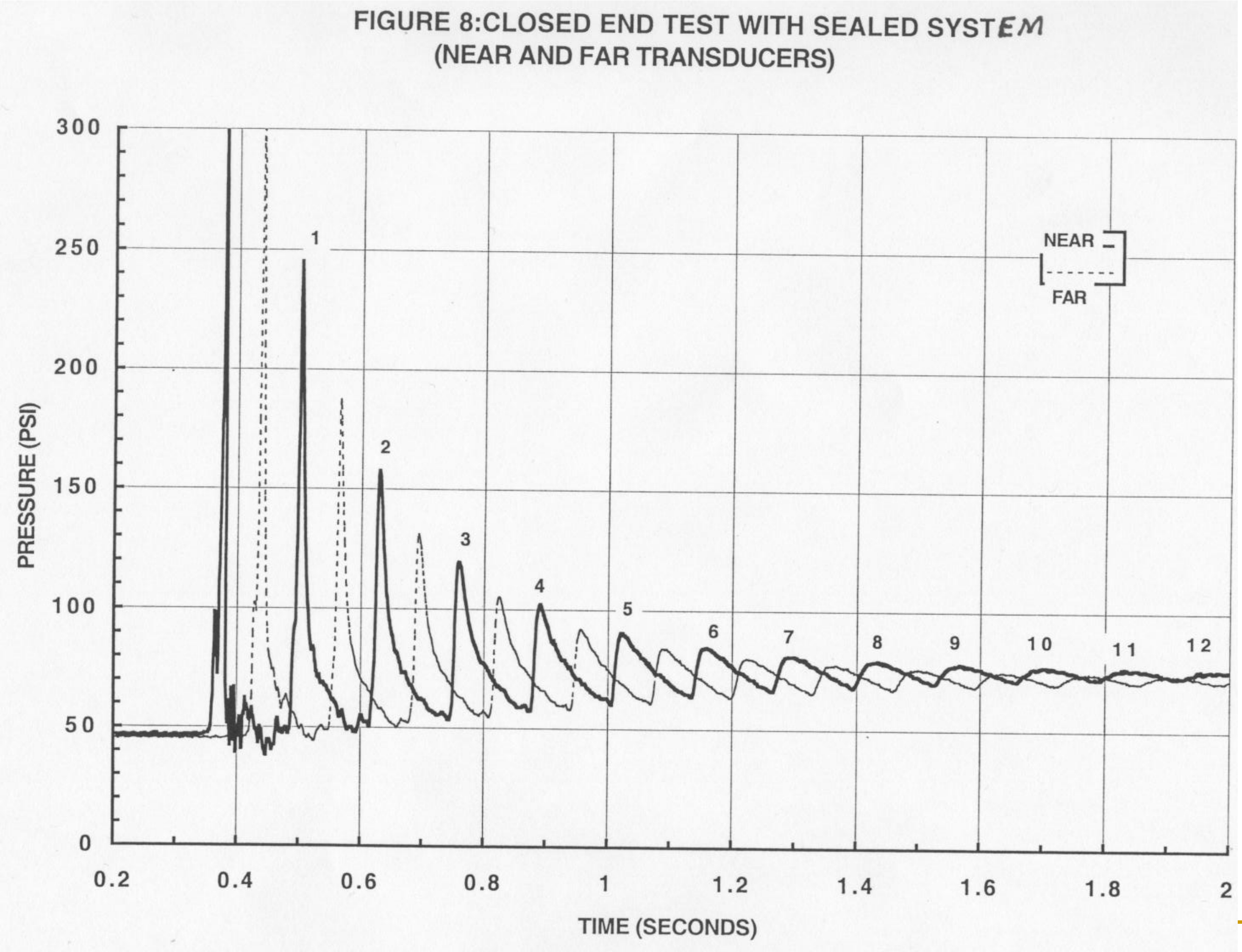


# Pinger

FIGURE 4: PINGER DESIGN



# Pressure Pulse in Small Diameter Tubing



# 1500 Foot Pulse Test